

Total No. of Questions : 8]
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**B.A./B.Sc. (General) Vth Semester (0005)
Examination**

0444

MATHEMATICS

Paper : II

(Modern Algebra)

Time : 3 Hours]

[Maximum Marks : 30

Note :- Attempt five questions in all, selecting at least two questions from each Section. All questions carry equal marks.

Section-A

1. (a) Let G be a finite group and $a \in G$ be an element of order n . Then $a^m = e$ iff n is a divisor of m .
- (b) If H is a sub-group of G of index 2 in G , then prove that H is normal subgroup of G .

A-330

(1)

Turn Over

2. (a) Let N be a normal sub-group of a group G . Show that G/N is abelian iff for all $x, y \in G$, $xyx^{-1}y^{-1} \in N$.

(b) Let H and K be two subgroups of a group G . Show that :

$$(H \cap K)C = H_C \cap K_C$$

for all $C \in G$.

3. (a) Let G and G' be two groups and $f : G \rightarrow G'$ homomorphism of G onto G' . Then prove that $\frac{G}{H} \cong G'$ where H is the kernel of f .

(b) If $f : G \rightarrow G$ defined by $f(x) = x^3 \forall x \in G$ is an automorphism, show that G is abelian.

4. (a) If $O(G) = p^2$, where p is prime number then prove that G is an abelian group.

(b) Prove that A_4 has no subgroup of order six, A_4 is alternative group of order 4.

A-330

(2)

Section-B

5. (a) If in a ring R , $x^3 = x$ for all $x \in R$, then show that R is commutative.
- (b) Let R be a finite ring without zero divisors and $O(R) > 1$. Then show that R is a division ring.
6. (a) Prove that intersection of two left (or right) ideals of a ring is a left (or right) ideal.
- (b) Prove that a commutative ring with unity is simple iff R is a field.
7. (a) Show that any ideal of \mathbb{Z} is maximal iff it is generated by some prime element.
- (b) Show that the mapping $f : \mathbb{Z} \rightarrow M_2(\mathbb{Z})$, defined by :

$$f(r) = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$$

is a homomorphism of rings, find $\ker f$. Is the mapping an isomorphism ?

A-330

(3)

Turn Over

8. (a) Let R be a commutative ring with identity and $f(x) \cdot g(x) \in R[x]$. Then $\deg.[f(x) g(x)] \leq \deg. f(x) + \deg g(x)$.

(b) For any ring R , show that $R[x] \langle x \rangle \cong R$.

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